THE MODEL APPROACH TO SYNCHRONIZATION

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At the present time, the Booster beam should have a bunching factor in phase space of between 0.1 and 0.2 prior to extraction. With an rf frequency of approximately 50 mHz, a period of $16\text{-}18 \times 10^{-9}$ sec is available to turn on the extraction kicker magnets.

In the future, it might prove desirable to achieve a better phase space match between Booster and Main Ring (M.R.) to reduce the voltage requirements on the M.R. cavities. This improvement could be accomplished by means of phase jumps to increase the bunching factor to ~0.5 or more. The result, however, would reduce the kicker turn on aperture to less than 10^{-8} sec. This figure may be improved, if necessary, by dropping one bunch from the booster at injection. During beam transfer, the extraction kickers can be synchronized to turn on as this empty bucket goes by. This adds an additional 20×10^{-9} sec for kicker turn on at the expense of a reduction of 1/84 in. beam intensity.

With this in mind, our synchronization approach should meet the following criteria:

1. Dilution of phase space area must be held to a minimum.

2. A maximum phase shift of $\pm 1/8$ Booster circumference or $\pm 21\ \pi$ rf radians must be possible should an empty bucket be introduced in the future.

The model comparison approach described here provides a flexible means for meeting these requirements under a variety of conditions.

First, let us define an interval of time (τ) before extraction in which both the frequency difference between Booster and M.R. (Δf) and its time derivative go from $\Delta f(0)$ and $\dot{f}(0)$ to zero in a programmed manner. During this interval, both the difference frequency and its time derivative can be defined to provide the proper phase slip to bring the frequencies into synchronization. Such a program can be carried out by letting:

$$\dot{f} = \frac{Lt^3}{3} + \frac{Mt^2}{2} - Kt - \dot{f}(0). \tag{1}$$

It follows that

$$\Delta f = \Delta f(0) + \int_0^t \dot{f}\, dt, \tag{2}$$

and

$$\Delta\phi = \Delta\phi(0) - \int_0^t \Delta f\, dt, \tag{3}$$

where $\Delta\phi = 2\pi n$ (revolutions). Values may be found for K, L, and M by solving Eqs. 1, 2, and 3 at $t = \tau$ where $\Delta\phi = 0$, $\Delta f = 0$, $\dot{f} = 0$ with initial conditions $\dot{f}(0)$, $\Delta f(0)$, and $\Delta\phi(0)$. Some measure of flexibility stems from the fact that the phase shifting program is independent of the period τ, therefore an array (K, L, M) exists for all t > 0.

In practice, both $\Delta f(0)$ and $\dot{f}(0)$ can be obtained from the rf system by direct measurements. $\Delta\phi(0)$, however, must be found by letting L = 0 in (1), then solving for K and M with (1) and (2) as before. $\Delta\phi(0)$ can then be found from (3).

The results of two sample phase shifting programs are plotted with the radial aperture (expressed in terms of frequency) as defined between the F and D magnets with a beam having an emittance of 100 π mm mrad at injection. See Figure 1. In the two cases illustrated here, $\Delta\phi(0)$ has been selected to cause the maximum rotation of + and - 21 π radians of the Booster rf in 8 msec. The area enclosed between the two curves is 42 π radians which represents 1/4 Booster revolution.

In practice, components in the model comparison approach to synchronization may be grouped into the following categories as illustrated in Fig. 2.

1. Frequency measurement
2. Model generation
3. Comparison

Frequency Measurement

The frequency difference between the Booster and M.R. is compared with a reference $\Delta\dot{f}(0)$. When the two are equal, a command is given to the Δf measuring circuits to measure the period of two consecutive cycle groups of phase detector no. 1

output. From these two readings, $f(0)$ can be computed. To provide sufficient resolution, a clock frequency near 5 MHz can be used. With a quarter cycle grouping (0 to $\pi/2$) this will provide a resolution of

$$R = \Delta f(0)^2/f(\text{clock}) = 72 \text{ Hz} \tag{4}$$

for a $\Delta f(0)$ of 100 kc. Each measurement for this case would require 52 μsec. Measurements are stored in local registers and can be used to update the model when necessary.

Model Generation

The model program as described by Eqs. 1, 2, and 3 can be carried out when given only the arbitrary parameters τ and $\Delta f(0)$. $\Delta f(0)$ is read into both the local frequency comparator and the model generation program. From the frequency measuring system data, $f(0)$ is calculated. $\Delta\phi(0)$ is then computed as described earlier, then modified to provide the closest number of quarter revolutions

$$\phi(\text{modified}) = 21 \text{ [Integer value } (\phi(0) + 10.5)/21]. \tag{5}$$

This modified value of $\phi(0)$ is then used with $\Delta f(0)$, $f(0)$ and τ to compute the coefficients of the modeling equations. The modeling program then generates a discreet value of Δf for each revolution of ϕ. These values are stored in a memory local to the equipment.

The modeling program is initiated when the following conditions are met:

1. Frequency difference between Booster and M.R. is less than $\Delta f(0)$
2. Phase detector (1) output at 0 radians.

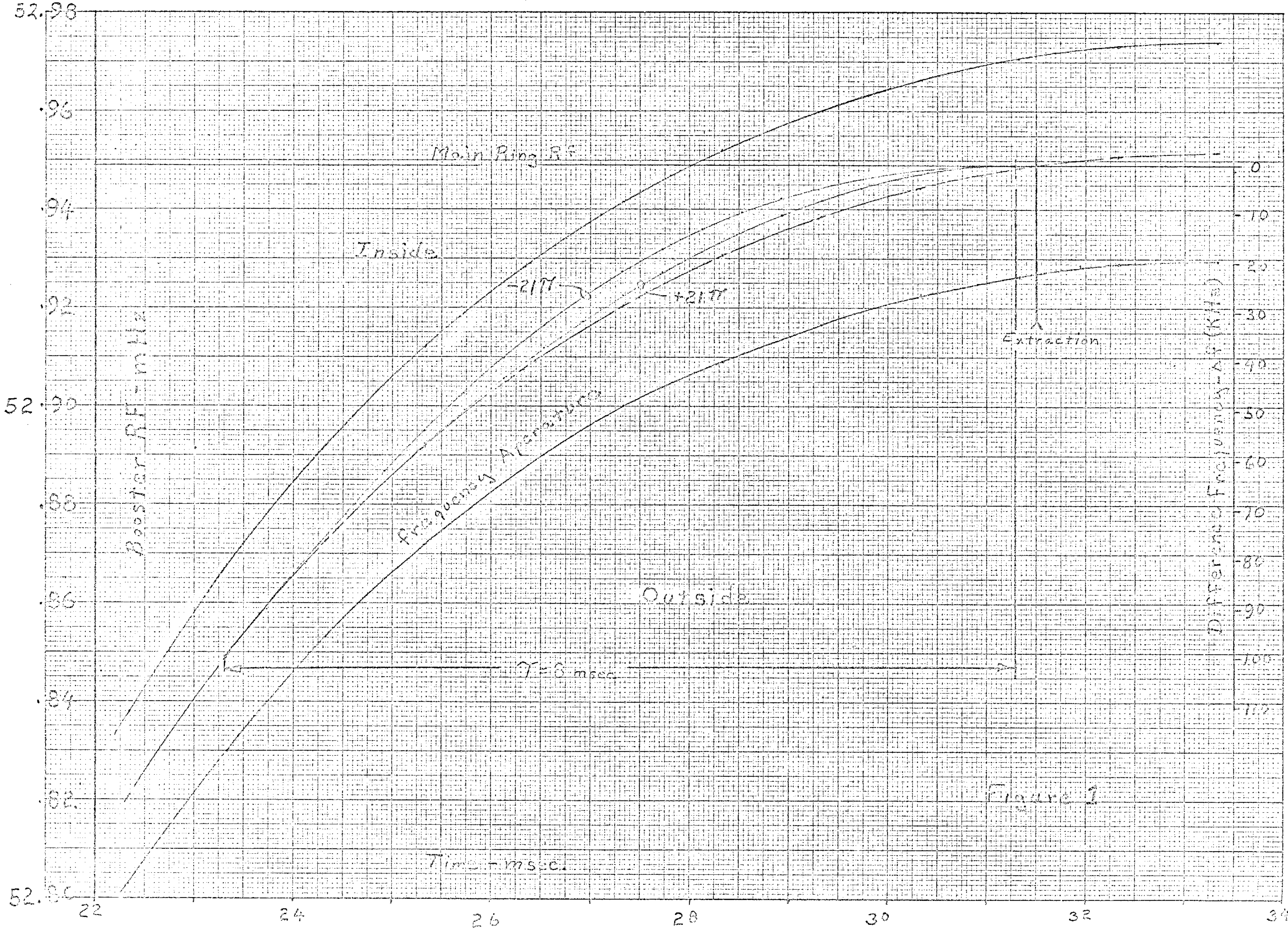
Once initiated, the first value is read out of memory into a D/A converter. This voltage is integrated to produce a voltage proportional to phase. Each time ϕ goes through one revolution as seen by phase detector 2, the next consecutive value of f is transferred into the D/A converter from memory. When the final value ($\Delta f = 0$) is reached, the model output will be zero.

3. Comparison

In order to fully appreciate the comparison approach, we should be aware of the fact that the phase detector output normally follows cos ϕ. The characteristic sign change requires that appropriate timing signals be generated by the phase detector to provide a corresponding sign change in the model. With appropriate circuitry, a linear relationship can be established between this output and ϕ. It follows, therefore, that the model output will have the same characteristic response as the phase detector output. This being true, we can compare the two outputs to produce a correction voltage for the Booster rf to force the actual frequency difference program to track the model. It is important to note that for the duration of the modeling program, the booster rf is phase locked to the main ring through the model. When the final value of Δf is read from memory ($\Delta f = 0$), the two frequencies are in phase and the synchronization is complete. Although this modeling approach does produce discreet frequency steps, the magnitude will be around 2 parts in 10^5 which should not contribute significantly to phase space dilution.

Sample Model Program

To illustrate the modeling approach, a computor program was written to simulate the Booster Frequency program. When the difference between M.R. and Booster frequencies becomes less than $\Delta f(0)$, the modeling program is initiated. In this example, a modeling period of 8 msec was used. Two runs were made--the first with $\Delta f(0) = 100.2$ kHz produces a total slippage $\phi(0)$ of 262.4 rf revolutions. This is modified to 252 rf revolutions which correspond to 12-quarter revolutions of the booster circumference. Increasing Δf to 100.3 kHz produces a slippage of 262.8 revolutions which is modified to 273 rf revolutions or 13-quarter revolutions of the Booster circumference. The program, having found the necessary initial conditions, computes the model coefficients K, L, and M, from these, the frequency, its derivative, and the slippage (in revolutions) has been computed as a function of time. Note that at the end of the program, all three parameters are zero.



52.98
.96
.94
.92
52.90
.88
.86
.84
.82
52.80
22
24
26
28
30
32
34
0
10
20
30
40
50
60
70
80
90
100
110
Booster RF - mHz
Main Ring RF
Inside
Outside
-2π
$+2\pi$
Frequency Aperature
Extraction
T=8 msec
Time - msec.
Difference Frequency - Δf (KHz)

Figure 1

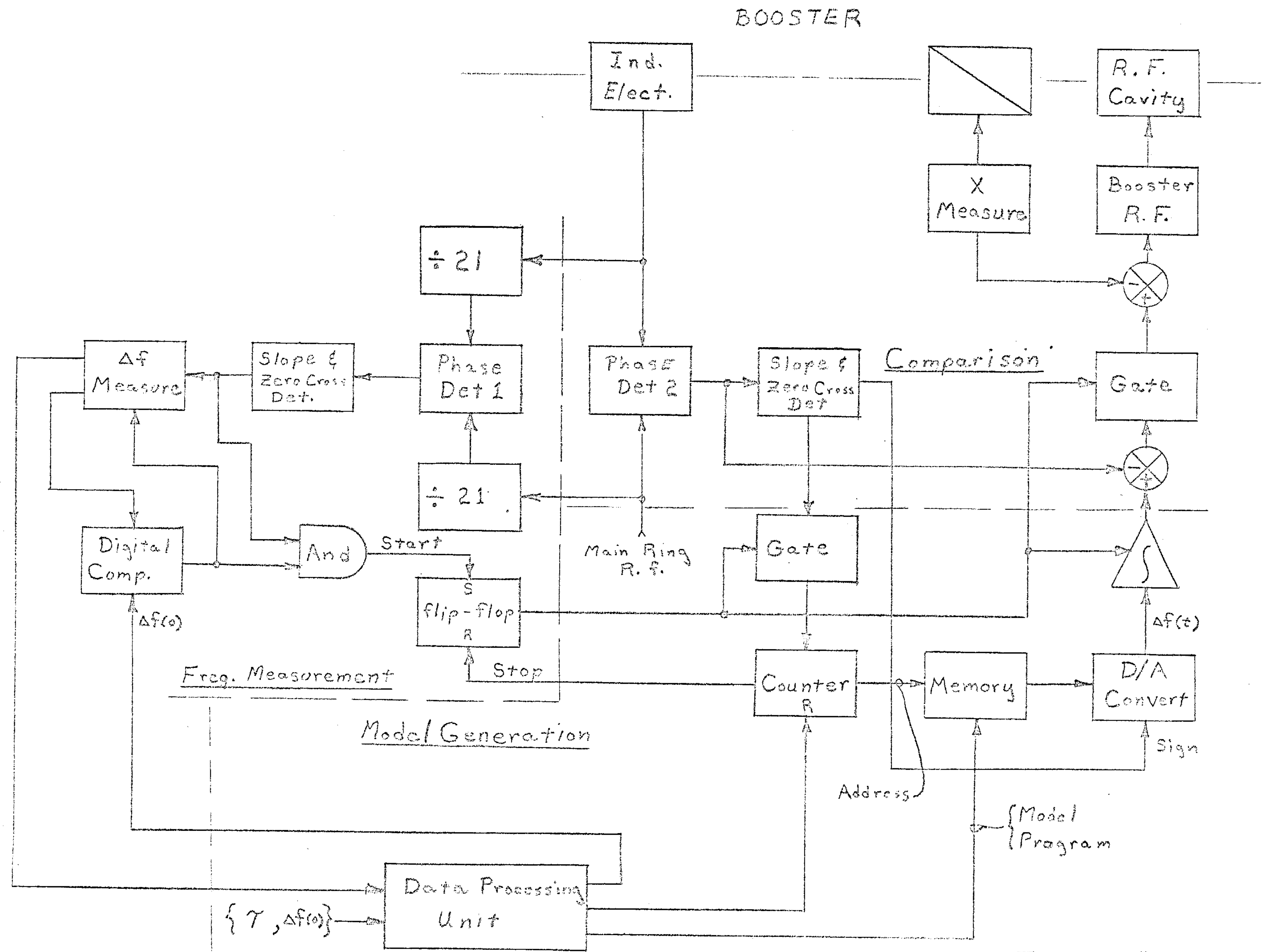

BOOSTER
Ind. Elect.
R. F. Cavity
X Measure
Booster R. F.
÷ 21
÷ 21
Phase Det 1
Phase Det 2
Slope & Zero Cross Det.
Slope & Zero Cross Det
Comparison
Gate
Gate
Δf Measure
Digital Comp.
And
Start
Stop
S
flip-flop
R
Δf(o)
Main Ring R. f.
Δf(t)
Counter
R
Memory
D/A Convert
Sign
Address
Model Program
Freq. Measurement
Model Generation
Data Processing Unit
{τ, Δf(o)}

Figure 2

AUTOTK 9:32 CH-C WE 05/28/8

MAIN RING R. F.- 5.29490E+07
TRANSITION PERIOD - SEC - .008
INITIAL DIFFERENCE FREQUENCY- HZ- 100200.00
STARTING FREQUENCY - 5.28492E+07
STARTING FDOT - 2.59454E+07
STARTING TIME - .0233
COMPUTE INITIAL VALUES FOR M AND K
INITIAL M - -8.39430E+10
INITIAL K - -3.57895E+09
PHI - 268.4246
PHI - INTEGER VALUE - 252

L	M	K
-1.14528E+14	8.32280E+11	-2.35732E+09

TIME	FDOT	F	PHI
.00	-2.59454E+07	100200.00	252.00
4.00000E-04	-2.49383E+07	90019.061	213.9696
8.00000E-04	-2.38128E+07	80265.139	179.9278
.0012	-2.25833E+07	70982.703	149.6946
.0016	-2.12647E+07	62210.359	123.0736
.002	-1.98716E+07	53980.85	99.8539
.0024	-1.84186E+07	46321.055	79.8129
.0028	-1.69204E+07	39251.987	62.7183
.0032	-1.53917E+07	32788.80	48.3305
.0036	-1.38470E+07	26940.779	36.4052
.004	-1.23012E+07	21711.348	26.6954
.0044	-1.07687E+07	17098.066	18.9539
.0048	-9.26437E+06	13092.63	12.9359
.0052	-7.80277E+06	9680.8711	8.4007
.0056	-6.39859E+06	6842.7576	5.1147
.006	-5.06647E+06	4552.3933	2.8534
.0064	-3.82108E+06	2778.0195	1.4039
.0068	-2.67708E+06	1482.012	.5672
.0072	-1.64912E+06	680.8845	.1603
.0076	-751879.09	145.2847	.019
.008	-.3437	-.0012	2.86102E-06

AT LINE NO. 400: STOP END

```
RAN     9476 SEC
120 DLTF=1.00385
RUN



AUTOTR      9:37    CH-C .. 05/22/78


MAIN RING R. F.-   5.29490E+07
TRANSITION PERIOD - SEC -              .008
INITIAL DIFFERENCE FREQUENCY- HZ-   100300.00
STARTING FREQUENCY -     5.28492E+07
STARTING FDOT -    2.59454E+07
STARTING TIME -               .0233
COMPUTE INITIAL VALUES FOR M AND K
INITIAL M -   -8.15992E+10
INITIAL K -   -3.56957E+09
PHI -         262.8246
PHI - INTEGER VALUE -         273
```

L	M	K
1.11791E+14	-9.75923E+11	-4.76200E+09

TIME	FDOT	F	PHI
.00	-2.59454E+07	100300.00	273.00
4.00000E-04	-2.41163E+07	90292.632	234.9059
8.00000E-04	-2.24290E+07	80988.064	200.6782
.0012	-2.08693E+07	72332.423	170.0289
.0016	-1.94227E+07	64277.559	142.7262
.002	-1.80751E+07	56781.045	118.5324
.0024	-1.68121E+07	49806.179	97.2318
.0028	-1.56194E+07	43321.983	78.6221
.0032	-1.44827E+07	37303.20	62.5182
.0036	-1.33876E+07	31730.299	48.7201
.004	-1.23199E+07	26589.472	37.0704
.0044	-1.12653E+07	21872.636	27.392
.0048	-1.02094E+07	17577.43	19.5161
.0052	-9.13791E+06	13707.216	13.2734
.0056	-8.03659E+06	10271.082	8.4925
.006	-6.89108E+06	7283.8379	4.9968
.0064	-5.68708E+06	4766.0185	2.6028
.0068	-4.41028E+06	2743.8811	1.1179
.0072	-3.04638E+06	1249.4077	.3374
.0076	-1.58105E+06	320.303	.043
.008	-1.4062	-.0034	6.67572E-06